and instead of (21) we have

$$S(l,s)=1+\sum_{n=1}^{N}\frac{\beta_n e^{-(l-\alpha_n)\xi}}{l-\alpha_n}-\frac{ig}{\sqrt{s}}\sum_{n=1}^{N}P_{n-1}\left(1+\frac{m_1^2}{2s}\right)\times\frac{e^{-(l+n)\xi}}{l+n}+\frac{ig}{\sqrt{s}}Q_l\left(1+\frac{m_1^2}{2s}\right),$$

since

$$S(l,s)=1+2i\sqrt{s}A(l,s)\,. \tag{21'}$$

In Eq. (5′) and (21′) $\beta_n(s)$ are now the residues of the partial-wave S matrix rather than the partial-wave amplitude.

In Fig. 1 (a)–(l), a plot of the real and imaginary parts of the S matrix versus the number of terms in the expansion for the Khuri series as well as for our series (21) for both $m=m_1$ and $m^2=4m_1^2$ is given. The horizontal lines correspond to the actual values of the S matrix. The Regge parameters used in the series as well as the actual S-matrix values have been calculated by numerical integration of the Schrödinger equation.

The fact that for $g=5$ the agreement is not quite as good as for $g=1.8$ may be due to small errors in the residues. For stronger potentials our numerical calculation of the residues is less accurate. And for $g=5$ it turns out that, in some cases, only a few percent error in the residues introduces a considerable error in the values of the real or imaginary parts of the S-matrix calculated from the series.

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Pseudoscalar and Vector Exchanges in the Production of Vector Mesons*

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The cross section for the general reaction (pseudoscalar meson +nucleon → spin-$\frac{1}{2}$ baryon+vector meson) and the decay angular distributions for the final baryon and vector meson are calculated under the assumption that the reaction is dominated by the exchange of pseudoscalar and vector mesons. The results are applied to an analysis of the reaction $\pi^- p\to\Sigma^0 K^{*0}$.

I. INTRODUCTION

In this paper the cross section for the general reaction (pseudoscalar meson+nucleon → spin-$\frac{1}{2}$ baryon +vector meson) is calculated by assuming that the reaction is dominated by the exchange of pseudoscalar and vector mesons. In Sec. II, we derive expressions for this cross section, and for the decay angular distributions for the final baryon and vector meson. Section III contains a discussion of the structure of the form factors that appear in these expressions. In Sec. IV we use the results of the preceding sections in an analysis of the reaction $\pi^- p\to\Sigma^0 K^{*0}$, which analysis is an extension of one reported earlier.[1]

II. CALCULATION OF CROSS SECTIONS

We will use the conventions $\hbar=c=1$, $g^{\mu\nu}=(1,\,-1,\,-1,\,-1)$, $A_\mu B^\mu=A^0B^0-\mathbf{A}\cdot\mathbf{B}$, $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}$, $\sigma^{\mu\nu}=\frac{1}{2}i\times[\gamma^\mu,\gamma^\nu]$, and $\gamma_5=\gamma^0\gamma^1\gamma^2\gamma^3$. Also, $\epsilon_{\mu\nu\lambda\sigma}$ is a completely antisymmetric tensor, which is $+1$ when $(\mu\nu\lambda\sigma)$ is an even permutation of (0123), -1 when it is odd, and zero otherwise. All spinors will be normalized so that $\sum_r u_r(p)\bar{u}_r(p)=p_\mu\gamma^\mu+m$.

Let us begin by considering the reaction $K^-p\to\Lambda\omega\to(\pi^-p)(\pi^+\pi^-\pi^0)$. Other reactions of the general form (pseudoscalar meson+nucleon → spin-$\frac{1}{2}$ baryon +vector meson) will have the same results, except for a possible over-all numerical factor for isotopic spin and a possible modification for different decay interactions for the final particles. Let p, r, H, Q be the momenta of the target nucleon, incident pseudoscalar meson, final baryon, and vector meson, respectively. Define two additional momenta, $k=H-p=r-Q$ and $s=p+r=H+Q$, so that k^2 and s^2 are the squares of the invariant momentum transfer and of the total center-of-mass energy, respectively. Let m, $\bar{m}$ be the masses of the target nucleon and incident pseudoscalar, M, $\bar{M}$ the masses of the final baryon and vector particle, and ν_p and ν_v the masses of the exchanged pseudoscalar (K) and vector (K^*) mesons, respectively. Then the most general Feynman amplitudes that can be written

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[1] Gerald A. Smith, Joseph Schwartz, Donald H. Miller, George R. Kalbfleisch, Robert W. Huff, Orin I. Dahl, and Gideon Alexander, Phys. Rev. Letters **10**, 138 (1963).

for the relevant three-particle vertices are

$$\mathfrak{M}(pK\Lambda)=(4\pi)^{1/2}\bar{u}(H)[f_0\gamma_5]u(p)\,, \tag{1a}$$

$$\mathfrak{M}(pK^*\Lambda)=(4\pi)^{1/2}\bar{u}(H)[f_1\gamma_\delta+if_2\sigma_{\delta\lambda}k^\lambda/(M+m)]u(p)e^\delta(k)\,, \tag{1b}$$

$$=(4\pi)^{1/2}\bar{u}(H)[(f_1+f_2)\gamma_\delta-f_2(p_\delta+H_\delta)/(M+m)]u(p)e^\delta(k)\,, \tag{1c}$$

$$\mathfrak{M}(KK\omega)=(4\pi)^{1/2}f_3r_\mu e^\mu(Q)\,, \tag{1d}$$

and

$$\mathfrak{M}(KK^*\omega)=(4\pi)^{1/2}f_4\epsilon_{\mu\gamma\sigma\rho}e^\mu(Q)e^\gamma(k)r^\sigma Q^\rho/\bar{M}\,, \tag{1e}$$

where $e(k)$ and $e(Q)$ are the unit polarization vectors of the exchanged and final vector particles, respectively. The five form factors f_i are dimensionless functions of the single variable k^2, which may be interpreted as the square of the "effective mass" of the exchanged particle.

The K- and K^*-exchange contributions then give a $K^-p\rightarrow\Lambda\omega$ Feynman amplitude

$$\mathfrak{M}(pK\Lambda\omega)=4\pi\bar{u}(H)[N_\mu e^\mu(Q)]u(p)\,, \tag{2}$$

where

$$N_\mu=F_0\gamma_5r_\mu+[F_1\gamma_\delta-F_2(p_\delta+H_\delta)/2M]\times(-g^{\delta\gamma}+k^\delta k^\gamma/\nu_v^2)\epsilon_{\mu\gamma\sigma\rho}r^\sigma Q^\rho/\bar{M} \tag{3a}$$

$$=F_0\gamma_5r_\mu+[F_1\gamma^\delta-F_2H^\delta/M]\epsilon_{\delta\sigma\mu\rho}r^\sigma Q^\rho/\bar{M} \tag{3b}$$

with

$$F_0=f_0f_3/(k^2-\nu_p^2)\,, \tag{4a}$$

$$F_1=(f_1+f_2)f_4/(k^2-\nu_v^2)\,, \tag{4b}$$

and

$$F_2=[2M/(M+m)]f_2f_4/(k^2-\nu_v^2)\,. \tag{4c}$$

If the ω and Λ polarizations are unobserved and the proton is unpolarized, then the

$$K^-p\rightarrow\Lambda\omega\rightarrow(\pi^-p)(\pi^+\pi^-\pi^0)$$

differential cross section in the c.m. system is

$$\sigma_H\equiv\frac{\partial\sigma}{\partial\Omega_{H,s}}=\frac{\eta|\mathbf{H}_s|}{64\pi^2s^2|\mathbf{r}_s|}\times\tfrac{1}{2}\sum_{p,\Lambda,\omega}|\mathfrak{M}(pK\Lambda\omega)|^2 \tag{5}$$

$$=\frac{\eta|\mathbf{H}_s|}{2s^2|\mathbf{r}_s|}\times\tfrac{1}{4}\,\mathrm{Tr}[(H_\alpha\gamma^\alpha+M)N_\mu\times(p_\beta\gamma^\beta+m)\bar{N}_\nu](-g^{\mu\nu}+Q^\mu Q^\nu/\bar{M}^2)\,. \tag{6}$$

We obtain $\bar{N}$ from N by replacing the F_i by their complex conjugates $\bar{F}_i$. The symbols $A_B{}^\mu=(A_B{}^0,\mathbf{A}_B)$ and $\Omega_{A,B}$ refer to the vector A and the direction of its spatial part, both evaluated in the "rest frame" of the vector B, i.e., the frame in which the spatial part of B vanishes. Thus the subscripts s, Q, and H refer to the c.m. system, the ω rest frame, and the Λ rest frame, respectively. The subscripts to the summation indicate which particles are included in the polarization sum. Note that we have included a factor $\eta\equiv\eta_Q\eta_H$ in the cross section, where $\eta_Q(\eta_H)$ is the fraction of the final vector particles (baryons) that decay via the observed decay mode.

Rather than evaluate σ_H now, we first investigate the decay distributions predicted for the final particles. For the decay $\omega\rightarrow\pi^+\pi^0\pi^-$, the most general I-spin-conserving Feynman matrix element is

$$\mathfrak{M}(\omega\pi\pi\pi)=f_5e^\nu(Q)q_+{}^\alpha q^\beta q_-{}^\gamma\epsilon_{\nu\alpha\beta\gamma}\,, \tag{7}$$

where q_+, q, and q_- are the π momenta, and f_5 is a function of two variables [e.g., q^0 and $(\Delta\equiv q_+{}^0-q_-{}^0)$] which is completely symmetric in the three π's. The $\omega\rightarrow\pi^+\pi^0\pi^-$ decay rate for a polarized ω is, in the ω rest frame,

$$\Gamma_\omega=\int\frac{d^3\mathbf{q}_-d^3\mathbf{q}d^3\mathbf{q}_+\delta^4(Q-q_+-q-q_-)}{(2\pi)^5 16\bar{M}q_+{}^0q^0q_-{}^0}|\mathfrak{M}(\omega\pi\pi\pi)|^2 \tag{8}$$

$$=\int\frac{d\Omega n,Qd\phi dq\text{-}d\Delta}{32(2\pi)^5M}|\bar{M}f_5\mathbf{q}_+\times\mathbf{q}_-|^2(\mathbf{e}_Q\cdot\mathbf{n}_Q)^2\,, \tag{9}$$

where $\mathbf{n}_Q$ is a unit vector in the direction $q_+\times q_-$ (i.e., normal to the decay plane) and ϕ is the azimuthal angle of $\mathbf{q}$ about $\mathbf{n}_Q$. Note that throughout this section the vectors q_+, q, and q_- are to be evaluated in the ω rest frame although the subscript Q has been suppressed.

If the target proton is unpolarized and the Λ polarization is unobserved, then the

$$K^-p\rightarrow\Lambda\omega\rightarrow(\pi^-p)(\pi^+\pi^-\pi^0)$$

differential cross section is

$$\sigma_{H,n}\equiv\frac{\partial\sigma_H}{\partial\Omega_{n,Q}}=A\frac{\eta|\mathbf{H}_s|}{64\pi^2s^2|\mathbf{r}_s|}\int\frac{d\phi dq^0d\Delta}{32(2\pi)^5\bar{M}}\times\tfrac{1}{2}\sum_{p,\Lambda}|\sum_\omega\mathfrak{M}(pK\Lambda\omega)\mathfrak{M}(\omega\pi\pi\pi)|^2 \tag{10}$$

$$=\frac{\eta|H_s|}{2s^2|\mathbf{r}_s|}\times\tfrac{1}{4}\,\mathrm{Tr}[(H_\alpha\gamma^\alpha+M)\mathfrak{A}]\times A\int\frac{d\phi dq^0d\Delta}{32(2\pi)^5\bar{M}}|\bar{M}f_5\mathbf{q}_+\times\mathbf{q}_-|^2\,, \tag{11}$$

where

$$\mathfrak{A}=(N_\mu n^\mu)(p_\beta\gamma^\beta+m)(\bar{N}_\nu n^\nu)\,. \tag{12}$$

Here A is a normalization factor to be chosen such that $\int d\Omega_{n,Q}\sigma_{H,n}=\sigma_H$, and n^μ is a covariant unit vector with the value $n_Q{}^\mu=(0,\mathbf{n}_Q)$ in the ω rest frame. From Eq. (11), we see that all azimuthal orientations of the ω decay plane about $\mathbf{n}_Q$, as measured by ϕ, are equally likely. Note also that the distribution in q^0 and Δ (i.e., the Dalitz-plot density) is independent of n^μ. Both these results are consequences of the form of the matrix element $\mathfrak{M}(\omega\pi\pi\pi)$, however, and cannot be used for an experimental check of our K- and K^*-exchange model. In treating the ω decay in this way, we have assumed only that the ω has a long enough lifetime—and thus decays sufficiently far from the Λ—so that its decay pions do not interact with the Λ. Use of the relation $\int d\Omega_{n,Q}n^\mu n^\nu=(4\pi/3)(-g^{\mu\nu}+Q^\mu Q^\nu/\bar{M}^2)$, along with a comparison of Eqs. (6), (9), and (11), shows $A=1/\Gamma_\omega$ and

$$\sigma_{H,n}=\frac{3\eta|\mathbf{H}_s|}{8\pi s^2|\mathbf{r}_s|}\times\tfrac{1}{4}\,\mathrm{Tr}[(H_\alpha\gamma^\alpha+M)\mathfrak{A}]. \quad (13)$$

Finally, we consider the $\Lambda\to\pi^-p$ decay, with a Feynman matrix element

$$\mathfrak{M}(\Lambda\pi p)=\bar{u}(h)[a+b\gamma_5]u(H), \quad (14)$$

where the constants a and b are the parity-nonconserving and parity-conserving amplitudes, respectively, and h is the decay proton momentum. In the Λ rest frame, a Λ of polarization $\mathbf{P}$ will correspond to

$$u(H)\bar{u}(H)=(1/2)M(1+\gamma^0)(1+\mathbf{P}\cdot\boldsymbol{\sigma})=(1/2)M(1+\gamma^0)\times(1+i\mathbf{P}\cdot\boldsymbol{\gamma}\gamma_5).$$

If the polarization of the decay proton is unobserved, then the $\Lambda\to\pi^-p$ decay rate in the Λ rest frame is

$$\Gamma_\Lambda=\int d\Omega_{h,H}\sum_p|\mathfrak{M}(\Lambda\pi p)|^2|\mathbf{h}_H|/32\pi^2M^2 \quad (15)$$

$$=\int d\Omega_{h,H}(1+\alpha\hat{h}_H\cdot\mathbf{P})C|\mathbf{h}_H|/16\pi^2M, \quad (16)$$

where

$$C=[|a|^2(h_H^0+M)+|b|^2(h_H^0-M)], \quad (17)$$

$$\alpha=2|\mathbf{h}_H|\,\mathrm{Im}(ab^*)/C, \quad (18)$$

and $\hat{h}_H$ is a unit vector in the direction of $\mathbf{h}_H$.

With unpolarized target protons and unobserved decay-proton polarization, the

$$K^-p\to\Lambda\omega\to(\pi^-p)(\pi^+\pi^-\pi^0)$$

differential cross section is

$$\sigma_{H,n,h}\equiv\frac{\partial\sigma_{H,n}}{\partial\Omega_{h,H}}=B\frac{3\eta|\mathbf{H}_s|}{8\pi s^2|\mathbf{r}_s|}\cdot\frac{|\mathbf{h}_H|}{32\pi^2M^2}\times\tfrac{1}{4}\,\mathrm{Tr}[\mathfrak{A}\mathfrak{B}], \quad (19)$$

where

$$\mathfrak{B}=(H_\alpha\gamma^\alpha+M)(\bar{a}+\bar{b}\gamma_5)(h_\beta\gamma^\beta+m)(a+b\gamma_5)\times(H_\delta\gamma^\delta+M). \quad (20)$$

As before, B is a normalization factor to be chosen such that $\int d\Omega_{h,H}\sigma_{H,n,h}=\sigma_{H,n}$. A simple calculation gives

$$\mathfrak{B}=2MC(1+i\alpha\gamma_5\hat{h}^\mu\gamma_\mu)(H_\alpha\gamma^\alpha+M)\equiv 2MC\mathfrak{D}, \quad (21)$$

where $\hat{h}^\mu$ is a covariant unit vector with the value $\hat{h}_H{}^\mu=(0,\hat{h}_H)$ in the Λ rest frame. A comparison of Eqs. (13), (16), and (19) then shows $B=1/\Gamma_\Lambda$ and

$$\sigma_{H,n,h}=\frac{3\eta|\mathbf{H}_s|}{32\pi^2s^2|\mathbf{r}_s|}\times\tfrac{1}{4}\,\mathrm{Tr}[\mathfrak{A}\mathfrak{D}]. \quad (22)$$

The trace calculation is straightforward, and yields

$$(1/4)\,\mathrm{Tr}[\mathfrak{A}\mathfrak{D}]=D_0+\alpha\hat{h}_H\cdot(\mathbf{D}_1+\mathbf{D}_2+\mathbf{D}_3), \quad (23)$$

where

$$D_0=\tfrac{1}{2}[(M-m)^2-k^2][|F_0|^2(\mathbf{r}\cdot\mathbf{n})^2+|F_1|^2(\mathbf{r}\times\mathbf{n})^2] +2[|F_1-\tfrac{1}{2}(1+m/M)F_2|^2-k^2|F_2|^2/4M^2]\times(\mathbf{H}\times\mathbf{r}\cdot\mathbf{n})^2, \quad (24)$$

$$\mathbf{D}_1=\mathrm{Im}[\bar{F}_0F_1][(M-m)^2-k^2](\mathbf{r}\cdot\mathbf{n})\mathbf{R}_H, \quad (25a)$$

$$\mathbf{D}_2=2\,\mathrm{Im}[\bar{F}_0(F_1-F_2)](\mathbf{r}\cdot\mathbf{n})(\mathbf{H}\times\mathbf{r}\cdot\mathbf{n})\mathbf{p}_H, \quad (25b)$$

and

$$\mathbf{D}_3=2\,\mathrm{Im}[F_1\bar{F}_2](\mathbf{H}\times\mathbf{r}\cdot\mathbf{n})(\mathbf{R}_H\times\mathbf{p}_H). \quad (25c)$$

Note that $\mathbf{H}$, $\mathbf{r}$, and $\mathbf{n}$ are to be evaluated in the ω rest frame, although the subscript Q has been suppressed for conciseness. R is a covariant vector with the value $R_Q=(0,\mathbf{r}\times\mathbf{n})$ in the ω rest frame. We obtain σ_H and $\sigma_{H,n}$ by simply integrating Eq. (22) with respect to the solid angles $\Omega_{h,H}$ and $\Omega_{n,Q}$.

The "rest frames" of s, Q, and H are related by Lorentz transformations without spatial rotations. In order to avoid any possible ambiguities, those spatial vectors whose values in the Λ rest frame are required in Eqs. (23) and (25) are given below, expressed in terms of the vectors $\mathbf{H}$, $\mathbf{r}$, $\mathbf{n}$, and $\mathbf{h}_Q$:

$$\mathbf{p}_H=-\mathbf{r}-\mathbf{H}[\bar{M}-r_Q{}^0+(\mathbf{H}\cdot\mathbf{r})/(H_Q{}^0+M)]/M, \quad (26a)$$

$$\mathbf{R}_H=(\mathbf{r}\times\mathbf{n})+\mathbf{H}(\mathbf{H}\times\mathbf{r}\cdot\mathbf{n})/M(H_Q{}^0+M), \quad (26b)$$

and

$$|\mathbf{h}_H|\hat{h}_H=\mathbf{h}_Q-\mathbf{H}[h_H{}^0+(\mathbf{H}\cdot\mathbf{h}_Q)/(H_Q{}^0+M)]/H_Q{}^0. \quad (26c)$$

Note that $h_H{}^0$ and $|\mathbf{h}_H|$ are constants, independent of $\mathbf{H}$ and $\mathbf{h}_Q$.

For other reactions we may have to modify the above expressions because of isotopic-spin considerations. The isotopic-spin dependence of the matrix elements in Eqs. (1) has not been exhibited because it results in a factor of unity. For the reaction $\pi^-p\to\Lambda K^{*0}$, however, we would include factors of $\bar{V}_KV_p$, $\bar{V}_{K^*}V_p$, $\bar{V}_{K^*}\tau_\pi V_K$, and $\bar{V}_{K^*}\tau_\pi V_{K^*}$ in Eqs. (1a), (1b), (1c), (1d), and (1e), respectively. Here V and $\bar{V}$ are two-component isotopic spinors, and τ is the isotopic-spin analog of the Pauli spin matrices. Thus the above cross sections would be multiplied by an over-all factor

$$|\bar{V}_{K^*}\tau_\pi V_p|^2=\left|(0,1)\begin{pmatrix}0&0\\\sqrt{2}&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right|^2=2.$$

Similarly, there would be a factor 2 for the reaction $\pi^-p\to\Sigma^0K^{*0}$, and a factor 4 for a reaction such as $\pi^+p\to\Sigma^+K^{*+}$. For the reaction $K^-p\to\bar{K}^0n$, which involves $\pi^\pm$ and $\rho^\pm$ exchange, there is a factor 4. The factor is unity for the noncharge-exchange reactions, $K^\pm p\to K^\pm p$, but since η and ω exchange are coherent with π^0 and ρ^0 exchange in these reactions, we must make the substitutions $F_0 \to F_{0\eta} \pm F_{0\pi}$, $F_1 \to F_{1\omega} \pm F_{1\rho}$, and $F_2 \to F_{2\omega} \pm F_{2\rho}$, where the sign $\pm$ is the same as the K charge, and where $F_{0\eta}$, $F_{0\pi}$, $F_{1\omega}$, $F_{1\rho}$, $F_{2\omega}$, and $F_{2\rho}$ each have the form given by Eq. (4). Note that reactions with an incident π will generally not involve both ρ and ω exchange, however, nor both η and π exchange, because this would not conserve G parity.

Finally, we consider the modifications to the above expressions that are necessary when particles other than the ω and Λ are produced. The matrix element for decay of the vector particle can always be written in the form $e^\mu(Q)V_\mu$. The amplitude for production and decay would then have the form $\bar{u}(H)[N_\mu v^\mu]u(p)$, where $v^\mu = (-g^{\mu\nu} + Q^\mu Q^\nu/\bar{M}^2)V_\nu$. Thus $v_Q{}^\mu = (0, -\mathbf{V}_Q)$, and we obtain the correctly normalized cross sections simply by redefining $\mathbf{n}$ as the unit vector in the direction $-\mathbf{V}_Q$. For $\rho \to \pi\pi$ or $K^* \to K\pi$, V^μ is just the momentum of one of the decay particles, so that $\mathbf{n}$ is thus a unit vector in the decay direction in the rest frame of the decaying particle.

The results for the Λ decay distribution apply directly to the $\Sigma^\pm$ decay, but with different experimental values for α. The situation is more complicated for spin-$\frac{1}{2}$ baryons that undergo a sequence of decays. The weak decay at the end of the sequence is the only decay that can produce anisotropy in the decay distributions, and is hence the only decay that can give any information on the polarization of the baryon. We will not consider such cases further, except for one comment concerning the $\Sigma^0 \to \Lambda + \gamma$ decay. It is predicted theoretically[2] that after integration over the photon direction and polarization, the average Λ polarization is one-third that of the Σ^0, but in the opposite direction. Therefore our previous results also apply to Σ^0 decay if we use a value $\alpha(\Sigma^0) = -(1/3)\alpha(\Lambda)$, and still let $\hat{h}_H$ be the direction of the decay proton in the Λ (not Σ) rest frame. Note that with our convention (i.e., protons emitted preferentially in the direction of the baryon spin for $\alpha > 0$), experimental values for α are[3] $\alpha(\Lambda) = 0.62 \pm 0.07$, and therefore $\alpha(\Sigma^0) = -0.21 \pm 0.02$.

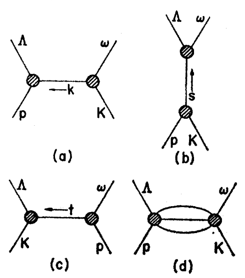
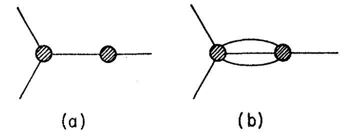


FIG. 1. Diagrams that contribute to the reaction $\pi p \to \Lambda\omega$.

[2] N. Byers and H. Burkhardt, Phys. Rev. **121**, 281 (1961).

[3] U. W. Cronin and O. E. Overseth, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 453.

FIG. 2. Typical diagrams that make up a three-particle vertex.



III. DISCUSSION OF THE MODEL

Again let us consider the reaction $K^- p \to \Lambda\omega$ as an example of our general reaction. The Feynman diagrams for this reaction may be separated into four sets of diagrams. The first set, represented by Fig. 1(a), contains all those diagrams that can be divided into two parts (one connected to the external p and Λ lines and the other to the external K^- and ω lines) by the cutting of a single internal line. In other words, these are merely the diagrams that contain a single virtual particle at some point in the k channel. Similarly, the second and third sets of diagrams, represented in Figs. 1(b) and 1(c), are those diagrams containing a single particle at some point in the s and t channels, respectively, where $t = H - r = p - Q$. The fourth set, represented by Fig. 1(d), contains all diagrams that cannot be so divided by cutting a single line, i.e., that contain two or more particles in each of the three channels.

Our model includes only those diagrams of the first set that represent the exchange of a vector or pseudoscalar particle. Scalar exchange cannot contribute because a scalar and pseudoscalar cannot be coupled to form a vector. We have not included the exchange of pseudovectors or of particles of spin greater than one, because we have not yet seen such particles experimentally. Diagrams of the other three sets are not included in the model. It is for reasons of mathematical convenience and simplicity, however, that these other diagrams have been ignored, rather than because of any good experimental evidence.

A more detailed look at the structure of the vertices in Fig. 1(a) will yield information on the form of the functions f_i. As seen in Eqs. (4), the functions F_i have poles at $k^2 = \nu^2$, where ν is the physical mass of the exchanged particle. Similarly, the form factors f_i have singularities corresponding to the physical masses of the various intermediate states in the vertices. Those vertex diagrams of the form of Fig. 2(a), where there is a single intermediate particle, would be expected to contribute simple poles. Diagrams of the form of Fig. 2(b), with two or more intermediate particles, will contribute branch cuts. If we use the Cauchy theorem and assume the proper convergence at infinity, then the branch-cut contribution to f_i may be written in the form $\int dz h_i(z)/(k^2 - z)$, which is merely an integration over a continuous distribution of poles. This is only to be expected, because a system of two or more particles has a mass that ranges continuously from some (positive) minimum up to infinity. Knowing the singularities of the form factor f_i, we may easily write down the form of the functions F_i. Note that the F_i have no

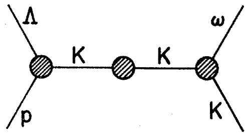


FIG. 3. This diagram gives a self-energy correction to the K propagator instead of producing a second-order pole in the scattering amplitude.

multiple poles. Multiple poles would correspond to a diagram such as Fig. 3, which merely gives a self-energy correction to the propagator, and does not (after renormalization,[4] at least) yield a second-order pole. Thus, where $\nu_0=\nu_p$ and $\nu_{1,2}=\nu_v$, we may write

$$F_i(k^2)=\frac{G_i}{k^2-\nu_i^2}+\int dz\frac{g_i(z)}{k^2-z}. \tag{27}$$

Note that from Eqs. (4) and (27) we have

$$G_0=2\pi i\,\mathrm{Res}[F_0,k^2=\nu_p^2]=f_0(\nu_p^2)f_3(\nu_p^2), \tag{28}$$

and similarly for the other G_i.

Since the form factors, f_i, evaluated on the mass shell, are just the corresponding renormalized coupling constants, we see that the G_i are merely products of the coupling constants at the two vertices. Reactions in which more than one vector particle can be exchanged (or more than one pseudoscalar) would have additional pole terms in Eq. (27), but these G_i would still be products of the coupling constants at the two vertices.

Let us briefly mention the qualitative effects to be expected from the other three sets of diagrams. Note that the functions F_i, from the first set of diagrams, are functions only of k^2 and are independent of s^2. Similarly, the other diagrams would introduce additional terms into the cross section, involving functions of s^2 only (from the second set of diagrams), of t^2 only (third set), and of s^2 and k^2 both (fourth set). (Note, incidentally, that there are only two independent variables because of the identity $s^2+k^2+t^2=M^2+\bar{M}^2+m^2+\bar{m}^2$.) The functions from the second and third sets of diagrams would have, except for different variables, the same form as for Eq. (27). However, those from the fourth set have only the branch-cut term, because, by definition, these diagrams do not have any single-particle intermediate states. Now consider those diagrams of Fig. 1(d) in which the exchanged system of particles is in a $J^P=0^-$ or $J^P=1^-$ state. These diagrams will then give a scattering amplitude identical to that for the exchange of a single pseudoscalar or vector particle, except that the functions F_i would have only the branch-cut term. The branch point will be the same as for single-particle exchange [because the possible intermediate states in Fig. 1(d) are the same as those in Fig. 2(b)], but the discontinuity across the branch cut will now be s^2-dependent. Therefore, these diagrams can be included in our original calculations merely by replacing the functions $g_i(z)$ in Eq. (27) by $g_i'(z,s^2)=g_i(z)+g_i''(z,s^2)$, where $g_i''(z,s^2)$ is the contribution from the exchange of the pseudoscalar or vector systems of particles. Thus, the only effect of such diagrams is to introduce an s^2 dependence into the branch-cut terms in Eq. (27), and our results will now describe any pseudoscalar or vector exchanges, regardless of whether these are exchanges of single particles or of multiparticle systems.

[4] The question of the renormalizability of field theories with vector particles does not really arise here. The results in this paper are in accord with the S-matrix philosophy, even though some of the terminology in the discussions is reminiscent of field theory.

IV. APPLICATION TO $\pi^-p\to\Sigma^0K^{*0}$

In this section, we present details of a previous analysis[1] of the reaction $\pi^-p\to\Sigma^0K^{*0}$, and also extend that analysis in order to obtain information about the functions F_i. As reported earlier,[1] 209 events at incident-pion lab momenta of 2.17 and 2.25 BeV/c were attributed to $\pi^-p\to\Sigma^0K^{*0}\to\Sigma^0K^+\pi^-$, and were found to have decay angular distributions

$$\rho(\alpha)=1+(1.59\pm0.55)\cos^2\alpha, \tag{29a}$$

$$\rho(\beta)=1-(0.11\pm0.21)\cos^2\beta, \tag{29b}$$

and

$$\rho(\gamma)=1-(0.74\pm0.11)\cos^2\gamma. \tag{29c}$$

Here α, β, and γ are the angles, in the K^* rest frame, between the direction of the decay pion and the unit vectors $\hat{N}$, $\hat{r}$, and $\hat{r}\times\hat{N}$, respectively, where $\hat{r}$ is the incident-pion direction and $\hat{N}$ is the normal to the production plane. Because of the small number of events, the effects of a non-K^* background (<30%) were ignored, and no attempts were made to determine either the Σ^0 decay distribution or the dependence of the functions F_i upon k^2 and s^2 (i.e., upon the π^-p c.m. energy and production angle).

The appropriate cross section,

$$\sigma_n=\int\int\sigma_{H,n,h}d\Omega_{h,H}d\Omega_{H,s},$$

is then found from Eqs. (22) to (24) to be

$$\sigma_n=C_0\cos^2\beta+C_1\sin^2\beta+C_2\cos^2\alpha, \tag{30}$$

where

$$C_i=\int d\mu J_i, \tag{31}$$

$$J_{0,1}=1/2[(M-m)^2-k^2]|\mathbf{r}|^2|F_{0,1}|^2|\mathbf{H}_s|/s^2|\mathbf{r}_s|, \tag{32a}$$

and

$$J_2=2[|F_1-1/2(1+m/M)F_2|^2-k^2|F_2|^2/4M^2]|\mathbf{H}_s|^3|\mathbf{r}_s|/\bar{M}^2. \tag{32b}$$

Here μ is the cosine of the angle between the final K^* and the incident π in the c.m. system, and we have used the relation

$$|\mathbf{H}\times\mathbf{r}|^2=(1-\mu^2)|\mathbf{H}_s|^2|\mathbf{r}_s|^2s^2/\bar{M}^2.$$

An over-all factor of 2 has been included for isotopic-spin considerations, as discussed in Sec. II. We have set $\eta=2/3$, because this is the branching fraction predicted by isotopic-spin conservation, for the $K^*\to K^+\pi^-$ mode and because the corrections for the detection efficiency of the Σ^0 will be included in the experimental data.

In our analysis,[1] we fitted Eq. (30) to Eqs. (29b) and (29c) and treated the errors coherently by solving

$$\begin{cases} 2C_0-2C_1-C_2=-(0.11+0.21a)(2C_1+C_2)\,,\\ -C_0+C_1-C_2=-(0.74+0.11b)(C_0+C_1+C_2)\,, \end{cases}$$

where a, $b=0\pm1$. [Note that $C_{0,1,2}$ corresponds to $g_{0,1,4}$ of Ref. 1, but that Eq. (3d) of that reference is written incorrectly.] The solution is then

$$\frac{C_2}{C_0}=\frac{4-2(1.89-0.21a)(0.26-0.11b)}{(0.89-0.21a)(2.26-0.11b)}\approx1.50+0.41a+0.28b=1.50\pm0.50$$

and

$$\frac{C_1}{C_0}=\frac{(2.89-0.21a)(0.26-0.11b)}{(0.89-0.21a)(2.26-0.11b)}\approx0.37+0.06a-0.14b=0.37\pm0.15\,.$$

Because our total cross section is

$$\sigma=\int d\Omega_{n,Q}\sigma_n=(4/3)\pi(C_0+2C_1+C_2)\,,$$

we may use the above solution to obtain

$$C_0\approx(0.31-0.05a)3\sigma/4\pi\,,$$
$$C_1\approx(0.115-0.045b)3\sigma/4\pi\,,$$

and

$$C_2\approx(0.46+0.05a+0.09b)3\sigma/4\pi\,.$$

The measured total cross section[5] for the reaction $\pi^-p\to\Sigma^0K^+\pi^-$ is 65 ± 12 μb and 53 ± 9 μb for incident-pion lab momenta of 2.17 and 2.25 BeV/c, respectively. This gives a mean value of $\sigma=59\pm11$ μb at 2.21 BeV/c, resulting in

$$C_0=4.4\pm1.1\,\mu\text{b}\,, \tag{33a}$$
$$C_1=1.6\pm0.7\,\mu\text{b}\,, \tag{33b}$$

and

$$C_2=6.5\pm1.8\,\mu\text{b}\,. \tag{33c}$$

For convenience, we ignore the errors and use only the mean values for the C_i in the rest of this paper.

To determine the form factors, $f_i(k^2)$, we use Eqs. (4) and evaluate Eqs. (31) at $s=2.25$ BeV (2.21 BeV/c incident momentum) to obtain

$$J_0=(11.9\mu\text{b})\frac{(1-0.438\mu)^2(1-0.721\mu)}{(1-0.619\mu)^2}|f_0f_3|^2\,, \tag{34a}$$

$$J_1=(5.92\mu\text{b})\frac{(1-0.438\mu)^2(1-0.721\mu)}{(1-0.435\mu)^2}\times|(f_1+f_2)f_4|^2\,, \tag{34b}$$

and

$$J_2=\frac{1-\mu^2}{(1-0.435\mu)^2}[(22.4\mu\text{b})|f_1f_4|^2+(5.10\mu\text{b})(1-0.765\mu)|f_2f_4|^2]\,, \tag{34c}$$

where $k^2=(-1.032+0.790)$ BeV^2. By using Eqs. (31), (33), and (34), we find that average values for the form factors in the region of this experiment are $|f_0f_3|^2\approx0.19$, $|(f_1+f_2)f_4|^2\approx0.14$, and $|f_1f_4|^2+0.20|f_2f_4|^2\approx0.20$, the latter two of which yield roughly $|f_1f_4|^2\approx0.1$ to 0.2 and $|f_2f_4|^2\lesssim0.2$.

These average values give us very little information about the values of the form factors on the mass shell, however, unless we can determine the k^2 dependence of the f_i from the production angular distribution. In this reaction, the form factors have no single-particle poles, but only the branch cut from the multiparticle intermediate states, so that $f_i(k^2)=\int dzh_i(z)/(k^2-z)$. We will make the approximation $f_i(k^2)\propto1/(k^2-\Lambda_i^2)$ in the physical region for this experiment, which gives

$$F_0(k^2)\approx(G_0+H_0)(\nu_p^2-\Lambda_0^2)(\nu_p^2-\Lambda_3^2)/(k^2-\nu_p^2)(k^2-\Lambda_0^2)(k^2-\Lambda_3^2)\,, \tag{35}$$

and similarly for the other F_i. The parameters Λ_i may be interpreted as average masses of the intermediate states of the form factors. Note that G_0+H_0 is the value of f_0f_3 on the mass shell ($k^2=\nu_p^2$) in this approximation, which is to be distinguished from the true value G_0 in Eqs. (27) and (28). By rewriting Eq. (35) as a sum of poles, we see that this approximation is equivalent to approximating the branch-cut integral of Eq. (28) by a sum of three poles with mutually related residues,

$$\int dz\frac{g_0(z)}{k^2-z}\approx\frac{H_0}{k^2-\nu_p^2}+\frac{(G_0+H_0)(\Lambda_3^2-\nu_p^2)}{(\Lambda_0^2-\Lambda_3^2)(k^2-\Lambda_0^2)}+\frac{(G_0+H_0)(\Lambda_0^2-\nu_p^2)}{(\Lambda_3^2-\Lambda_0^2)(k^2-\Lambda_3^2)}\,. \tag{36}$$

The branch cut in F_0 begins at $(m_K+2m_\pi)^2=0.59$ BeV^2 (because $K+\pi$ cannot be in a 0^- state), which is well above the pole at $\nu_p^2=0.24$ BeV^2. Thus in F_0 the $H_0/(k^2-\nu_p^2)$ term has no direct physical meaning, and is introduced merely to improve the approximation in the physical region. It is expected that $H_0\ll G_0$,

[5] Gerald A. Smith, University of California, Lawrence Radiation Laboratory, Berkeley, California (unpublished).

which means that G_0+H_0 is a good approximation to G_0. For $F_{1,2}$, however, the branch cut begins at $(m_K+m_\pi)^2=0.40$ BeV^2, which is below the pole at $\nu_v^2=0.78$ BeV^2. The corresponding $H_{1,2}/(k^2-\nu_v^2)$ term thus approximates the lower part of the branch cut, so that $H_{1,2}$ is a measure of the coupling to the low-mass multiparticle intermediate states in $F_{1,2}$, just as $G_{1,2}$ gives the coupling to the K^*. In view of the small number of events in this experiment, let us make the further simplification of setting $\Lambda_0=\Lambda_3=\lambda$ and $\Lambda_1=\Lambda_2=\Lambda_4=\Lambda$. Thus our final approximation is

$$f_0f_3/\hat{g}_0=(\nu_p^2-\lambda)^2/(k^2-\lambda^2)^2 \tag{37a}$$

and

$$f_1f_4/\hat{g}_1=f_2f_4/\hat{g}_2=(\nu_v^2-\Lambda^2)^2/(k^2-\Lambda^2)^2, \tag{37b}$$

where $\hat{g}_0=G_0+H_0$, and similarly for $\hat{g}_{1,2}$.

When we combine Eqs. (31), (34), and (37) and compare the results with the experimental values in Eqs. (33), we obtain equations of the form

$$|\hat{g}_0|^2=A(\lambda), \tag{38a}$$

$$|\hat{g}_1+\hat{g}_2|^2=B(\Lambda), \tag{38b}$$

and

$$C(\Lambda)|\hat{g}_1|^2+D(\Lambda)|\hat{g}_2|^2=1. \tag{38c}$$

Since the relative phase of $\hat{g}_1$ and $\hat{g}_2$ is unknown, Eqs. (38) do not determine the magnitudes of $\hat{g}_1$ and $\hat{g}_2$, but only yield the restriction

$$\left|\frac{CB^{1/2}-(C+D-BCD)^{1/2}}{C+D}\right|\leqslant|\hat{g}_2| \leqslant\left|\frac{CB^{1/2}+(C+D-BCD)^{1/2}}{C+D}\right|, \tag{39}$$

with a similar restriction on $|\hat{g}_1|$ due to Eq. (38c). The production angular distribution, $\rho(\mu)=(J_0+2J_1+J_2)/(C_0+2C_1+C_2)$, therefore involves the three parameters λ, Λ, and $|\hat{g}_2|^2$. A maximum-likelihood fit to the experimental[1] $\rho(\mu)$ yields $\lambda=\infty$, $\Lambda=1.19_{-0.33}^{+0.27}$ BeV, and $|\hat{g}_2|^2=0.1_{-0}^{+85}$, with a χ^2 of 6.2 for six degrees of freedom. The upper and lower values for Λ are those for which the likelihood function L falls to $1/e$ of its maximum value $L_{\max}$. All values of $|\hat{g}_2|^2$ in the region $0.1<|\hat{g}_2|^2<85$, as allowed by Eq. (39) for $\Lambda=1.19$ BeV, correspond to values $L>L_{\max}/e$. The values for the other g_i are $|\hat{g}_0|^2=0.185$, $|\hat{g}_1+\hat{g}_2|^2=30$, and $27\geqslant|\hat{g}_1|^2\geqslant14$ for $\Lambda=1.19$ BeV and $0.1\leqslant|\hat{g}_2|^2\leqslant85$. The range of values for λ with this fit may be seen by noting that the values $\lambda=2.06(0.95)$ BeV, $\Lambda=1.46(3.3)$ BeV, $|\hat{g}_0|^2=0.56(9.4)$, $|\hat{g}_1+\hat{g}_2|^2=4.3(0.28)$, $|\hat{g}_1|^2=4.2$ (0.35), and $|\hat{g}_2|^2=0.001(0.004)$ correspond to $L=L_{\max}/e$ $(L=L_{\max}/e^2)$, with $\chi^2=8.5(10.2)$. The variations about the mean values in Eqs. (33) do not significantly alter these results.

Note that K and K^* exchanges do not contribute to the related reaction $\pi^-p\to\Sigma^-K^{*+}$, so that one would expect a considerably smaller cross section for this reaction than for $\pi^-p\to\Sigma^0K^{*0}$ is the K- and K^*-exchange model is valid. Experimentally, at 1.90 and 2.05 BeV/c incident-pion lab momenta, we have[5] $\sigma(\pi^-p\to\Sigma^-K^{*+})\approx30\%\sigma(\pi^-p\to\Sigma^0K^{*0})$. This indicates that other mechanisms are present, but the results are still consistent with the assumption that K and K^* exchange is the dominant mechanism for $\pi^-p\to\Sigma^0K^{*0}$.

The above values for $|\hat{g}_0|^2$ will yield values for $|f_0(\nu_p^2)|^2$, since the experimental K^* width can be used to determine $|f_3(\nu_p^2)|^2$. In this section, we have used the Feynman matrix element

$$\mathfrak{M}(\pi KK^*)=\bar{V}_{K^*}\tau_\pi V_K(4\pi)^{1/2}f_3r_\mu e^\mu(Q).$$

The total $K^{*0}\to K\pi$ decay rate is then

$$\Gamma=\frac{|\mathbf{r}|}{8\pi\nu_p^2}\times\tfrac{1}{3}\sum_{K^*,K,\pi}|\mathfrak{M}(\pi KK^*)|^2,$$

where $\mathbf{r}$ is the decay-pion momentum in the K^* rest frame, and the summation is over the K^* polarization and the K and π isotopic-spin indices. Thus,

$$\Gamma=|f_3(\nu_p^2)|^2|\mathbf{r}|^3/2\nu_p^2=(14.4\text{ MeV})|f_3(\nu_p^2)|^2.$$

By setting this decay rate equal to the experimental value of 50 MeV for the full width at half-maximum for the K^*, we obtain $|f_3(\nu_p^2)|^2=3.5$. We thus obtain $|f_0(\nu_p^2)|^2=|\hat{g}_0|^2/|f_3(\nu_p^2)|^2=0.053$, 0.16, and 2.7 for $\lambda=\infty$, 2.06, and 0.95 BeV, respectively. These values correspond to values 0.47, 0.45, and 0.29 (or alternatively to 0.53, 0.55, and 0.71) for the f parameter of Martin and Wali.[6]

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[6] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).